$$\ddot{\mathbf{b}}_0{}^k = \sum_{i=0}^{k=1} [\mathbf{A}_0{}^i \cdot \ddot{\mathbf{b}}^{i+1} + 2\boldsymbol{\omega}_0{}^i \times \mathbf{A}_0{}^i \cdot \dot{\mathbf{b}}^{i+1} +$$
$$\dot{\boldsymbol{\omega}}_0{}^i \times \mathbf{A}_0{}^i \cdot \mathbf{b}^{i+1} + \boldsymbol{\omega}_0{}^i \times (\boldsymbol{\omega}_0{}^i \times \mathbf{A}_0{}^i \cdot \mathbf{b}^{i+1})] \quad (5i)$$

$$\mathbf{I}_0{}^k = \mathbf{A}_0{}^k \cdot \mathbf{I}^k \cdot {}^T\mathbf{A}_0{}^k \quad (5j)$$

$$\dot{\mathbf{I}}_0{}^k = \boldsymbol{\omega}_0{}^k \times \mathbf{A}_0{}^k \cdot \mathbf{I}^k \cdot {}^T\mathbf{A}_0{}^k - \mathbf{A}_0{}^k \cdot \mathbf{I}^k \cdot {}^T\mathbf{A}_0{}^k \times \boldsymbol{\omega}_0{}^k \quad (5k)$$

with ${}^T\mathbf{A}_0{}^k$ being the transpose or conjugate of $\mathbf{A}_0{}^k$.

Example

We will assume a satellite P^0 on which is mounted an unbalanced 2-gimbal gyro with P^1 being the outer gimbal, P^2 the inner gimbal, and P^3 the rotor. A coordinate system $B^i(i = 0, 1, 2, 3)$ is attached rigidly to P^i at its center of mass. The dyadic $\mathbf{A}^i$ takes components in B^i to components in B^{i-1}; $\mathbf{A}^0 = \mathbf{E}$. The center of mass of P^i relative to B^{i-1} is $\mathbf{b}^i$ (the components are understood to be in B^{i-1} by our convention). The rotation of B^i $(i = 1, 2, 3)$ relative to B^{i-1}, with components known in B^{i-1}, is $\boldsymbol{\omega}^i$. It is desired to know the acceleration $\ddot{\mathbf{b}}_0{}^3$ of P^3 relative to P^0, with components in B^0, in terms only of the known quantities just given. By application of formula (5i), in conjunction with (5a, 5d, and 5g), we have

$$\ddot{\mathbf{b}}_0{}^3 = \ddot{\mathbf{b}}^1 + \mathbf{A}^1 \cdot \ddot{\mathbf{b}}^2 + 2\boldsymbol{\omega}^1 \times \mathbf{A}^1 \cdot \dot{\mathbf{b}}^2 + \dot{\boldsymbol{\omega}}^1 \times \mathbf{A}^1 \cdot \mathbf{b}^2 +$$
$$\boldsymbol{\omega}^1 \times (\boldsymbol{\omega}^1 \times \mathbf{A}^1 \cdot \mathbf{b}^2) + \mathbf{A}^1 \cdot \mathbf{A}^2 \cdot \ddot{\mathbf{b}}^3 +$$
$$2(\boldsymbol{\omega}^1 + \mathbf{A}^1 \cdot \boldsymbol{\omega}^2) \times \mathbf{A}^1 \cdot \mathbf{A}^2 \cdot \dot{\mathbf{b}}^3 +$$
$$(\mathbf{A}^1 \cdot \dot{\boldsymbol{\omega}}^2 + \boldsymbol{\omega}^1 \times \mathbf{A}^1 \cdot \boldsymbol{\omega}^2) \times \mathbf{A}^1 \cdot \mathbf{A}^2 \cdot \mathbf{b}^3 +$$
$$(\boldsymbol{\omega}^1 + \mathbf{A}^1 \cdot \boldsymbol{\omega}^2) \times [(\boldsymbol{\omega}^1 + \mathbf{A}^1 \cdot \boldsymbol{\omega}^2) \times \mathbf{A}^1 \cdot \mathbf{A}^2 \cdot \mathbf{b}^3] \quad (6)$$

We have in (6) a general relation but, as only relative rotation can occur, we can write some of the foregoing terms more explicitly. For example, let B^i rotate with respect to B^{i-1} about an axis $\hat{\mathbf{n}}^i$, with components in B^{i-1}, through an angle ϕ^i. Then $\boldsymbol{\omega}^i = \hat{\mathbf{n}}^i\dot{\phi}^i$ and $\dot{\boldsymbol{\omega}}^i = \hat{\mathbf{n}}^i\ddot{\phi}^i$ (time differentiation is with respect to B^{i-1}). Also, let $\mathbf{b}^i$ be made up of two parts; these are a displacement from B^{i-1} to that point on $\hat{\mathbf{n}}^i$ where the plane normal to $\hat{\mathbf{n}}^i$ passes through B^i (called $\mathbf{a}^i$) plus a displacement from this point to B^i (called $\mathbf{c}^i$). Thus $\dot{\mathbf{b}}^i = \boldsymbol{\omega}^i \times \mathbf{c}^i = \dot{\phi}^i\hat{\mathbf{n}}^i \times \mathbf{c}^i$ and $\ddot{\mathbf{b}}^i = \dot{\boldsymbol{\omega}}^i \times \mathbf{c}^i + \boldsymbol{\omega}^i \times (\boldsymbol{\omega}^i \times \mathbf{c}^i) = \ddot{\phi}^i\hat{\mathbf{n}}^i \times \mathbf{c}^i + (\dot{\phi}^i)^2\, \hat{\mathbf{n}}^i \times (\hat{\mathbf{n}}^i \times \mathbf{c}^i)$. At this point, on substitution into (6), we have all of the information on the right-hand side known as it would be in practice.

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NERVA Thermal and Fluid-Flow Analysis

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Introduction

This note discusses a digital computer program developed to accomplish over-all performance studies with respect to the thermal and fluid-flow analysis in the nuclear engine for rocket vehicle application (NERVA) program. In evaluating the performance of a system, the interactions between the component characteristics and the system behavior depends on the steady state behavior of the system and upon five major transient effects. These transient effects can be described by thermal, nuclear, pneumatic, acoustic, and mechanical response characteristics, and all of them need to be considered at some time in an analysis program. Limitations of time and money preclude the consideration of all of the pertinent effects in one analysis program for a complicated system such as NERVA and have led to the establishment of program analysis requirements based on separation of effects. Since acoustical and mechanical transients normally involve frequencies much higher than 1 cps, they would require much more computer time for complicated systems than slower thermal, nuclear, and pneumatic transients. Although the provision for nuclear transients is included in the computer program, this discussion will be limited to the thermal and pneumatic considerations; control-system characteristics are also excluded.

Heat-Transfer and Fluid-Flow Data

Early in the NERVA program it was recognized that data representing the thermal and hydraulic behavior of hydrogen were meager and needed to be supplemented before realistic use of a program, such as described in this note, could be made. Studies performed during the past several years, and proceeding on a continuing basis, have provided this information to a large degree. The correlations used in this program are of necessity quite flexible and are based on data from various sources.

The coolant heat-transfer correlation used is based on data[1,2] that formally separates the regions of "liquid," two-phase flow, and "gas," although in the supercritical region there is actually no distinction between the regions. The exhaust gas heat-transfer correlation is based on scale-model tests for nozzles of similar geometry.[3,4]

In nuclear rocket systems of the NERVA type, thermal radiation from the core to the surrounding surfaces is appreciable and is accounted for by suitable shape factors.[5] The nuclear heating aspects in the system are accounted for by approximations of more exact calculations.[6]

General Program Description

The system being simulated is described primarily in terms of thermal nodes, fluid-flow elements, and other elements. Each thermal node is specified in terms of the nature and amount of the solid material assumed to be centered at the thermal node and its share of the nuclear and gamma radiation-flux levels. Its current state is specified by the temperature. The initial temperature must be supplied for every thermal node.

In a given fluid element, the inlet and outlet cross-sectional areas for flow may be different, and the corresponding momentum change is allowed for.[7] The effective area for pressure-drop and heat-transfer coefficient calculations may differ from both of the other areas for that element. Additional loss coefficients are used to allow for classical friction pressure drops, surface roughness factor, entrance or exit losses, or valve characteristics.

The capacitance of the flow element is expressed as the volume available for storage of fluid, and this volume is assumed to contain fluid at the average of the inlet and outlet densities. The length of the flow element is specified along with the distance from the entrance L_e, to allow for boundary-layer growth and entrance effects. The tube can be representative of n similar tubes of identical geometry and conditions, and the term flow element applies to the group. Analytical flexibility in dealing with complex geometrical shapes has been obtained in the program, and coolant passages can be arranged in various series and/or parallel combinations.

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Other elements differ in characteristics from those assumed for fluid-flow elements. This classification includes items such as pumps and turbines. The hot gas side of the nozzle is also treated differently from fluid-flow elements. The changes in stagnation pressure and temperature were insignificant, so that only the rate of heat-transfer by convection to the adjacent surface nodes need be considered. Recent test data have shown that three-dimensional flow effects in the nozzle are important. Therefore, the results of detailed method of characteristics calculations were fitted by an empirical equation to expedite the calculations although retaining these three-dimensional effects.[7]

Thermal nodes may be connected by conduction and/or radiation connections. Any pair of nodes may be connected, and there can be several different connections between the same pair of nodes. For example, there could be a radiation connection and several conduction connections. Each conduction connection can have a fixed conductance Y, or it can be a function of the average of the two nodal temperatures. If the conduction path includes an interface (of two different metals, for example), which may introduce a significant contact resistance, the total resistance for the two material impedances plus the contact is computed from the sum of the individual resistances. The sum of all of the conductances and all of the heat inputs to each thermal node is computed.

Truncation Errors and Computational Experiments

In using an analysis technique that relies on a finite difference approximation of the differential equations governing the behavior of a distributed impedance system such as described in this paper, care must be taken to obtain a space and time representation that yields both a stable and accurate computational scheme. However, the spatial truncation errors can be a real problem in the analysis of a system, since too fine a grid requires both excessive computer memory and computational time, whereas a coarse grid can give inaccurate results.

There are three major types of truncation errors incurred in an analysis of this type. They are 1) fluid temperature errors, 2) pressure-drop errors, and 3) solid material temperature errors. Computational accuracy on a per node basis has been greatly improved by the use of the average fluid element temperature in place of the entering temperature. The pressure-drop formulation used minimizes errors caused by changes in fluid properties which are computed using empirical equations developed to minimize computer time required for adequate computational accuracy.[8]

The principal difficulty in the numerical solution of multidimensional problems of solid material temperatures arises from the treatment of nonuniform mesh sizes. The method adopted was previously developed[9] for a relaxation solution of La Place's equation. The implicit set of backward difference equations are presented in the preprint.[10]

The change in enthalpy and pressure in each fluid element is computed using a formula that includes fluid property values at the downstream face. An iterative procedure is used during which the entire length of the fluid passage is recalculated until the boundary conditions are satisfied. However, it is also necessary to recalculate, or loop, at each element in a given progression downstream. After a number of trial cases were computed, it was determined that five loopings at each node were sufficient for the first iteration, with successive reductions to two loopings in the fourth and following iterations. The first iteration is especially sensitive, because it begins with the use of previous time point values. Cases have been found where an instability occurs in the computation if no loopings are made. Table 1 shows the result of the first step in a nozzle chill-down calculation. The temperature upon exiting from the coolant tubes is seen to approach rapidly the true value when five loopings were made; the approach is made much more slowly and erratically with one loop. IBM 7094 computer times elapsed for four time points were 2.6 min for 1 loop, 7.3 min for 5 loops, and 3.9 min for the regressive reduction from 5 to 2 loops which scheme gave practically identical results for convergence as the 5-loop case.

Table 1 Outlet bulk temperature, °R

Iteration count	5 loops	1 loop
1	Initial guess = 162.00	
2	166.6	165.57
3	157.9	166.02
4	153.6	166.20
5	152.1	166.28
6	151.7	166.22
7	151.55	166.01
8	151.50	165.63
9	151.48	165.10
20	151.46	157.05

Table 2 Percent error between 6 node and 21 node model for representative nozzle heat-transfer and fluid-flow parameters for approximate full-power reactor conditions

Quantity	% error	Quantity	% error
Heat transferred from hot gas side, Btu/sec	−3.0	Max thermal flux of coolant (Btu/in.^{2} sec)	−27
Pressure drop in coolant tubes, psi	0.3	Max wall temperature, °R	−18
Rise in bulk temperature, °R	2.4	Max Mach number of coolant	−20

Computer time charged is also a function of the number of nodes and fluid elements. The use of 21, 11, and 6 elements to describe a certain version of the NERVA nozzle coolant tube during a chill-down transient showed negligible differences in computed results. The same number of nodes and other representation was kept for the inlet pipe, toroidal header pipe, etc., in all of the cases. In the full-power case, however, care must be taken so that the averaging effect inherent in using a smaller number of nodes does not give an erroneous indication of peak values. A good example of this may be seen in Table 2 where the peak values of temperature, heat flux per unit area, and mass flow have been appreciably changed, although the over-all temperature rise of the coolant, the total heat through the nozzle wall, and the total pressure drop are quite close, using either model. The maximum number of nodes used coincides with that found advisable in steady state calculations with similar configurations for which the results of calculations made over a period of several years were available.

Conclusion

Experience gained with the subject program shows that closed loop transient NERVA simulation can be analyzed with the use of two-phase hydrogen as coolant. This usefulness of the basic approach, trading flexibility in simulation for complexity in coding the basic program, has been proved for an operation that has a long continuing need for such a calculation. Flexibility has proved useful not only in dealing with various hardware changes and engine models, but also changes in level of complexity of simulation. Relatively simple models are adequate to describe the interaction between system components; however, more detailed models are used for certain component analyses and to correlate better the numerical results obtained by similar groups in industry.

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Water Recovery by Membrane Permeation

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RESEARCH into the problems concerned with the recovery of potable water from metabolic waste has progressed to that point wherein several techniques have been studied and evaluated. Reference 1 probably contains the most complete bibliography, description, and evaluation of possible methods for water recovery. Paraphrasing from this report, it is pointed out in the conclusions that the vacuum pyrolysis method in which potable water is produced from mixed human waste and wash water is operationally most advanced. This technique, which has been fully tested both chemically and biologically, was originally devised in this laboratory[2] and further improved under support of NASA.[3] It has certain advantages that other methods do not have. For example, the raw material may be processed immediately after collection with no prior treatment. Also, the recovered product is immediately acceptable in potability and does not require further treatment.

This particular process begins by vaporizing the volatiles in the waste, passing these vapors over a heated catalyst and vapor condensation, resulting in potable water. However, as in other systems, this particular process is hindered by the energy input. Not only is there an energy requirement for the change of state but also a small energy input in the catalytic zone in order that the catalyst be maintained at the proper temperature levels.[4] By proper design,[5] the over-all energy input can be reduced to that point wherein values of power requirements are actually less than the power required to vaporize water. However, problems in engineering design do exist because of the necessity of relatively high temperatures at which the catalyst must be maintained. Thus, the elimination of the catalyst would simplify the technique.

Permselective Membrane Processes

Kammermeyer[6] of the State University of Iowa discovered the un˙que high permeation rates of gases through silicone rubber. More recent interest in the use of permselective membrane has been renewed because of the availability of better membranes, having improved strength/thickness ratios and the need for inexpensive separation processes to fulfill the requirements in the fields of medicine and space.

Because different gases go through polymeric films with varying ease, it appeared theoretically possible to separate the water from the other constituents found in urine by vaporizing this water and permitting it to permeate through the membrane. Previous experiments had shown that there was no significant increase in the permeation rate of liquid water when it was placed on one side of the membrane and 500 psig were applied to it. However, the vapor, having a permeation rate orders of magnitude greater than the liquid, would pass through. Consequently, an experiment was derived to check this hypothesis.

Experimental

The experimental setup for this study is simple and consists merely of a still pot having a membrane located over its neck followed by a condenser and a receiver. The raw material, initially a pooled sample of undiluted urine, is placed in the still pot, and the entire system is then evacuated by connection to a vacuum pump. The system pressure is maintained at approximately 40–50 mm Hg (0.77–0.97 psia) where the volatiles in the urine will boil at approximately 40°C (104°F). A collection flow rate of 190 ml/hr (0.2 lb/hr) was maintained with a membrane having a surface area of 85 cm^2 (13.1 in.^{2}). The vapor is condensed and collected in the receiver. The collected product was submitted to Betz Laboratories, Philadelphia, Pa., for analysis. The results indicate that the material is potable and contains no odor or color. Figure 1 is an ultraviolet spectrophotometric analysis of distilled water, General Electric catalytic recovered water, Norristown water, and the product derived from the membrane system described previously. As can be seen, the material compares quite favorably with the distilled water reference. Both the "membrane water" and the "catalytic water" are far superior in purity to both the Philadelphia and Norristown water.

Additional experiments were conducted in which the raw material was composed of one part urine to four parts of detergent wash water solution and processed as described earlier. Here again, the recovered product was adjudged potable.

Although all actual experiments conducted with urine were made with one particular membrane composition, many other materials were tested. Polyvinyl alcohol (40.0) and cellulose

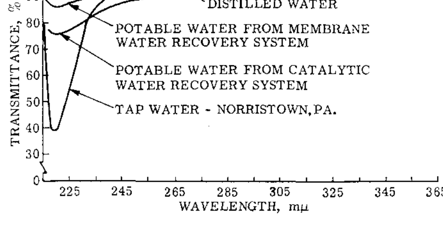

Fig. 1 Comparison of several water recovery methods by ultraviolet transmittance.

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